A study on the shallow water effect on a ship's pivot point

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Abstract

Information on the pivot point of a turning ship is collected, taking into account practical notes and manuals on ship maneuvering as well as experimental data and simulated results which all together reveal a consistent behavior when varying water depth or some ship particulars. Results from the studies already carried out on the Riverine Support Patrol Vessel (RSPV) of the Colombian Navy are included in this one, in order to estimate the pivot point's position and to contrast those results with theory and available empirical observations. Linear manoeuvrability theory is tested and its results show poor approximation with respect to the kinematic equations. As to the depth variation effect, by means of full-scale experiments it is confirmed that the pivot point's position, when going to shallow water, always varies in the same way, proving to be coherent with the available information on this phenomenon.

Keywords: pivot point, shallow water, manoeuvrability

Estudio de efectos por aguas someras sobre el punto de pivote de un buque

Resumen

Se recopila información relacionada con el punto de pivote (o punto de giro) del buque durante un giro, teniendo en cuenta información derivada de notas y manuales prácticos de maniobra de buques así como datos experimentales y numéricos que en conclusión dejan ver un comportamiento consistente al variar la profundidad del agua o algunas características del buque. Los estudios sobre un buque en particular, la nodriza o patrulla de apoyo fluvial pesada (PAF-P) son incluidos en este, con el fin de estimar el punto de pivote y confrontar los resultados con las teorías y observaciones empíricas documentadas. La teoría de maniobrabilidad lineal es probada y los resultados revelan una aproximación muy pobre con respecto a las fórmulas puramente cinemáticas. En cuanto al efecto de la profundidad, se comprueba con experimentación a escala real que la posición del punto de pivote, al pasar a aguas someras, varía siempre en el mismo sentido y es coherente con la información disponible de este fenómeno.

Palabras clave: punto de pivote, aguas someras, maniobrabilidad

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Introduction

The pivot point (PP) is a non-fixed point standing along the symmetry axis of a ship and that has no sideways motion while the ship is turning. In other words, it is the position on the symmetry axis that has a zero drift angle (Tzeng, 1998; Port Revel, 2011). According to Cauvier (2008), a more accurate name should be “apparent pivot point”. That point may be taken as a guide, for instance, for maneuvers with little available room (Tzeng, 1998) or tugging operations (Cauvier, 2008). For the first case it is convenient that the pivot point falls close to the center of gravity so that the swept area during turning is the least possible (Tzeng, 1998); while if tugging a ship, the farther from the center of gravity the pivot point is located, the more effective the lateral forces exerted by the tug are and the smaller the turn is (Cauvier, 2008). Given its usefulness for maneuvering practical issues, this topic is mainly addressed by publications oriented to ship captains or commanders. Nonetheless there exist as well kinematic definitions that allow this point’s position to be evaluated; and there are too dynamic theories which offer an estimate of the pivot point’s steady state location. These methods are useful tools to approximately assess the pivot point since the design stage (Tzeng, 1998).

From the available bibliographic sources it is possible to get a lot of information about the relationship between the pivot point’s position and the manoeuvrability characteristics under different conditions of operation, shape of the ship and its propulsion system. Also it is remarkable the existing emphasis on making clear the differences between PP and the centre of lateral resistance (COLR) which is taken as the point of leverage for the effective lateral forces, because in some references this difference is not clearly pointed out and the concepts handled in there could ambiguously mislead to the wrong conception that the PP is the centre of leverage for yaw moments. The PP is an effect of motion and it is not a property or something intrinsic of the ship (Cauvier, 2008); its position is a function of the lateral forces acting on the vessel, and this is why its location is not a fixed point (Port Revel, 2011). By extending this last concept, by assuring that the PP’s position varies as a function of the pressure fields around the ship, it comes clear that design features as geometry and propulsion devices, and external factors like restricted waters, both the velocity and the attitude of the ship, the interaction with other bodies and the action of wind and currents may modify the location of the PP.

As to the ship proportions, it is stated that a bulkier vessel and with a wider beam features a PP closer to the bow when moving ahead and performing a turn (Cauvier, 2008). This is observed along with a higher resistance from underwater forward of PP, phenomenon which is compensated by a more open drift angle (Hooyer, 1983).

As to the effects of depth on the manoeuvrability, it has been noticed that in shallow water there have been identified larger turning diameters, smaller drift angles and then a greater advance, a greater speed loss due to the resistance increase, squat, and it has been noticed that larger rudder angles are needed in order to achieve the same handling characteristics as in deep water manoeuvring (Southampton Institute, 2001; Sagarra, 1998). This behavior has been extensively reported and studied, and additionally it has been observed in the mathematical models of motion, being named as the standard effect (Yoshimura, 1988). The same variations were reported in other reference (Hooft, 1973), but adding a specific mention about the PP, which, according to the author, displaces backwards when shifting the depth from deep to shallow water, and therefore falling closer to the center of gravity. The explanation to this is provided by Cauvier (2008) by reminding that in shallow water the transversal force (lift) is larger than in deep water, and thereby the PP gets closer to the COLR, and thus the drift is smaller. From other point of view, in shallow water there is an increase of pressure abaft of PP and to avoid an excessive resistance the obtained drift angle is smaller (Hooyer, 1983).

When analyzing the propulsion system, it is clear that when a very effective lateral force is applied, a greater moment is generated and for the resistance to compensate it, a wide drift angle is produced (Cauvier, 2008). This phenomenon is seen, for example, on boats with off-board or jet propulsion, which allow a tighter turn and a greater drift angle than with a typical propeller-rudder system (U. S. Coast Guard, 2003).

As the main study material, it has been collected all the available information about the design, numerical models and experimental results carried out with the riverine support patrol vessel (RSPV) of the Colombian National Navy (Carreño, 2011). The RSPV is characterized for being a vessel with a bulky hull, with a high beam-draft ratio and for being thrusted by a pump-jet system, which consists of a pair of centrifugal pumps with a steerable discharge, located by the stern. A set of full-scale experiments was made with this ship having different initial speeds, water depth levels and jet angles in the propulsion system (Carreño et al, 2011). Furthermore a non-linear mathematical model was developed to do simulations in three degrees of freedom (DOF) of the manoeuvrability of this design based on the formulations proposed on diverse publications, as well as with test data of resistance to advance and self-propulsion carried out with a scale model (Carreño et al, 2012). The advantage of having this information available is to be able to assess the actual behavior of the PP with the full-scale results and thus to test the existing equations related to the PP position.

Dynamic concepts

The reference system, as well as the definition of the variables involved in the 3-DOF manoeuvrability model, is shown in figure 1. Sign convention is also established there, according to the direction of the plotted vectors (position, velocities and forces) and angles (drift β, heading ψ and jet/rudder δ) on the diagram.

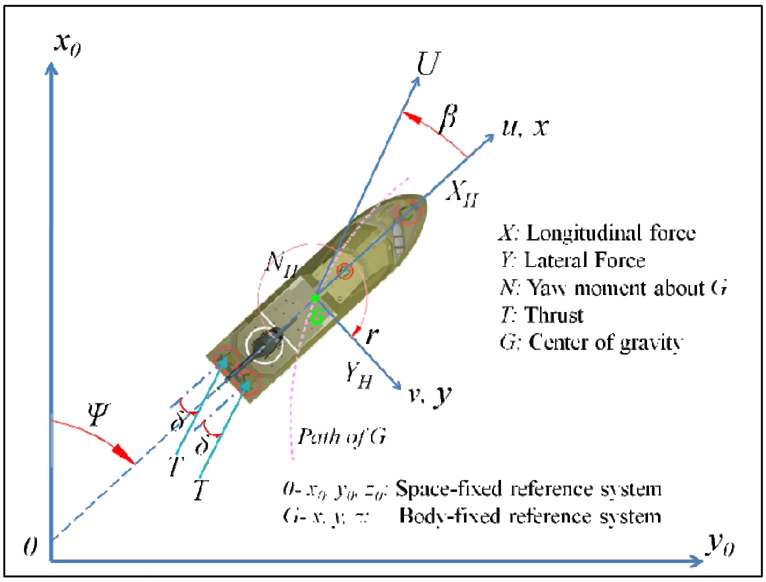

Figure 1. System of reference for manoeuvrability model with three degrees of freedom

Thank to the concepts presented by the MMG (Japanese Mathematical Modeling Group) in several works by some of its members (Inoue, 1981), (Lee & Kijima, 2006), and by other authors who have supported their models on those (Pérez & Clemente, 2007), the complete model for manoeuvrability that governs for the description of the surface ship motion is defined by the following equation system:

$$(m + m_x)\dot{u} - (m + m_y)vr + \frac{1}{2}(Y_{\dot{r}} + N_{\dot{v}})r^2 = X,$$

$$(m + m_y)\dot{v} - Y_{\dot{r}}\dot{r} + (m + m_x)ur = Y,$$

$$(I_{zz} + J_{zz})\dot{r} - N_{\dot{v}}\dot{v} - (m_x - m_y)uv + \frac{1}{2}(Y_{\dot{r}} + N_{\dot{v}})ur = N,$$

Equation in which the three DOF or the system unknowns are the two linear velocities u and v and the angular yaw velocity r. Variables m_x, m_y, J_{zz}, $Y_{\dot{r}}$ y $N_{\dot{v}}$ are added inertia terms which arise from the motion of an object in a fluid. On the other hand, variables X, Y and N are the exerted forces and the moment acting on the vessel, whose hydrodynamic part may comprise linear terms only or include non-linear components too.

By following the already shown convention, and according to the PP definition given above, one can proceed to do a kinematic analysis on this point. Then, let the local sideways velocity be denoted by $\dot{y}(x)$; this can be defined as follows:

$$\dot{y}(x) = v + rx$$

The meaning of this equation may be appreciated in figure 2, where the kinematic scheme of a turning circle maneuver is displayed. The pivot point, longitudinally located at x_p, produces a nil local sideways velocity; then:

$$\dot{y}(x) = 0 \; if \; x = x_p$$

Therefore:

$$0 = v + rx_p$$

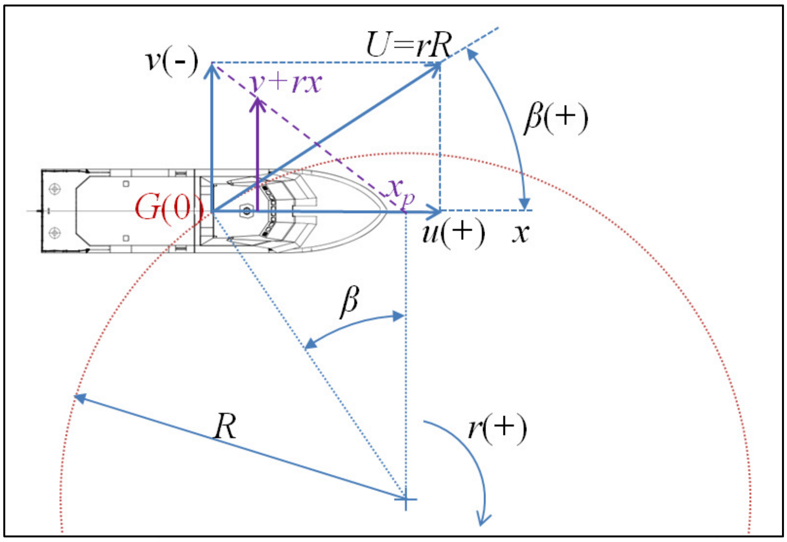

Figure 2. Kinematic scheme of a turning ship

Starting from this equation several formulae can be stated in order to calculate the PP by using the theory of the turning ship dynamics:

- **Formula I:** it is derived directly from the physical concept (Fossen, 2011; Tzeng, 1988) :

$$x_p = \frac{-v}{r}$$

- **Formula II:** it is obtained from the maneuver kinematics (Sagarra, 1998) (see Fig. 2):

$$x_p = \frac{-v}{r} = \frac{-(U\sin(-\beta))}{r}$$

$$x_p = \frac{U\sin\beta}{r}$$

$$x_p = \frac{rR\sin\beta}{r}$$

$$x_p = R\sin\beta$$

Formulae I and II are theoretically equivalent, but when counting on real data they might return different numerical results due to the likely measurement errors acquired in the experiments in addition to the acceleration that are still present at the time of measuring and that would lead to the loss of the equivalency between the two above formulae since they are deducted in the context of a steady motion condition.

- **Formula III:** it is deduced from linear manoeuvrability theory, assuming the steady state and applying this onto the second order Nomoto model (Fossen, 2011; Tzeng, 1988):

$$x_p' = -\frac{N_r' Y_\delta' - (Y_r' - m' u_0') N_\delta'}{Y_v' N_\delta' - Y_\delta' N_v'}$$

where Y_v', Y_r', N_v', N_r', Y_δ', N_δ' are the non-dimensional forms of the hydrodynamic linear derivatives (coefficients) associated to yaw and sway. The first four of them may be estimated by the using the geometric particulars of the vessel, by means of some equations obtained from by regression of data from several hull types. The other two derivatives are related to the effect of propulsion and the angle applied on this handling system (δ), which depend on the way that the propulsion forces and moments are mathematically modeled. In this model, the lateral component of propulsion (Y_P) is assumed to approximate to a linear expression as a function of angle δ like this:

$$Y_\delta \delta \cong Y_P$$

$$Y_\delta \cong \frac{Y_P}{\delta}$$

Supposing that the pumps at both starboard and port sides produce equal effects, the transversal propulsion force turns simplified (with respect to the one developed for the algorithm of the referred source) (Carreño, 2011), and so its associated coefficient yields:

$$Y_\delta = \frac{-2F_m(1-t)T_{j,0}\sin\delta}{\delta}$$

F_m is the maneuver factor (service factor of the pumps for the maneuver's initial speed), t is the thrust deduction, and $T_{j,0}$ is the nominal thrust generated by each pump, that varies in function of the ship speed and water depth during the maneuver. As to the yaw derivative N_δ includes only the moment caused by the transversal force Y_P, and does not take into account the moment that could appear because of the longitudinal components of propulsion, which would exist only if the thrust of starboard and port had a different magnitude.

$$N_\delta = Y_\delta x_j$$

where x_j is the longitudinal position of the discharge of the propulsion pumps. Having the above definitions as a basis, it is possible to eliminate these two derivatives from formula III, as it is possible to factorize $Y_\delta{}'$ off the expressions in both the numerator and denominator, and so making the formula free of propulsion specifications. That expression thus turns into:

$$x_p' = -\frac{N_r{}' - (Y_r{}' - m'u_0{}')x_j{}'}{Y_v{}'x_j{}' - N_v{}'}$$

Information about the studied vessel

The ship which has been taken as the object of analysis in this paper is the so named riverine support patrol vessel (RSPV), which has been developed by COTECMAR and serves the Colombian National Navy (Fig. 3). Its hull corresponds to a riverine ship with small deadrise and with a high beam-draft ratio, designed to sail on very shallow water; the ship profile plane is shown in fig. 4. The propulsion system is composed by a pair of *pump-jet* type centrifugal pumps, ref. SPJ 82RD, made by *Schottel*, powered by two MTU - series 60 diesel engines, which produce 450BHP at 1800RPM, and are coupled through a reduction and reverser gear along with a cardan shaft. The pump jet can be steered all over 360° individually or in tandem by means of a joy-stick control at the command bridge or locally at the engine control room.

Figure 3. Photograph of the RSPV during sea trials (Source: COTECMAR)

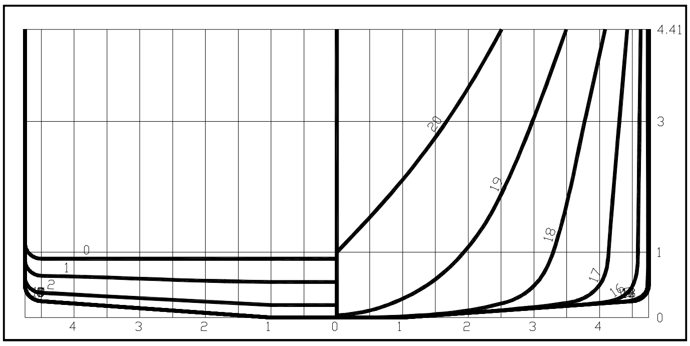

Figure 4. Hull profile draft (Source: COTECMAR)

Fort the calculation of the coefficients associated to manoeuvrability it is required to have specific information on the analyzed vessel and thereby some particulars of the RSPV are summarized and presented in table 1. The non-dimensional version of all of the involved variables in the model is the corresponding to the so named Prime system – II (Fossen, 2011).

Table 1. Geometric data of the RSPV

Variable	Value	Unit
m'	0.41116	non-dimensional
d	1	m
B	9.5	m
L	37.92	m
C_B	0.781	non-dimensional
x_j	-15.81	m

When running computationally the herein referred model, the linear coefficients proposed by Inoue (1981) were implemented and their value was evaluated considering the necessary vessel particulars. The used coefficients are the ones contained in table 2 whether the simulated case was deep water or shallow water. These hydrodynamic derivatives are the ones to be used for evaluating formula III.

Table 2. Hydrodynamic coefficients implemented in simulations and in formula III for PP

Coefficient	Coefficients according to Inoue's equations (with trim)	Correction factor by depth effect (h/d=2.2)	Coefficients for h/d=2.2
Y'_v	-0.6324	5.5884	-3.5341
Y'_r	0.0798	1.9614	0.1565
N'_v	0.002635	6.3075	0.01662
N'_r	-0.0346	1.5212	-0.0526

Results

Besides the information given above regarding the RSPV features, t was also possible to collect complete enough data from experimental tests on scale models of several types of ship having several values of water depth. In addition to the results of the calculations made on the RSPV, in table 5 there are registered the data extracted from the proper publications concerning the other vessels (Lee & Kijima, 2006) (Yoshimura & Sakurai, 1988), on which the motion parameters of every ship are reported for a maneuver at a constant rudder angle but at two different depth conditions. The output data therein presented are the yaw rate r, the steady state absolute speed U_s, the drift angle β and the steady turning radius R, along with the distances between G and the PP (x_p shown in length units and in non-dimensional form). As to the "ship C" taken from Lee & Kijima (2006) it was not possible to estimate its PP by applying formula III because the necessary coefficients were unknown. For all of the models reported here, x_p is measured with respect to G except the wide beam vessel (Yoshimura & Sakurai, 1988), whose position is measured from the midship O (G is located 3.39% of L forward of O).

Table 3. Evaluation of the PP with data of the RSPV and the scaled models of other vessels

Case	U_0 (m/s)	L (m)	r (rad/s)	U_s (m/s)	β (°)	R (m)	Formula I		Formula II		Formula III	
							x_p (m)	x_p'	x_p (m)	x_p'	x_p (m)	x_p'
PAF-P (full scale) δ=20° h/d=24 Starboard (Carreño, 2011)	4.63	37.9	0.058	0.92	77.8	8.87	27.7	0.732	19.7	0.519	93.1*	2.46
PAF-P (full scale) δ=20° h/d=2.2 Starboard (Carreño, 2011)	4.32	37.9	0.072	1.54	74.9	9.07	20.6	0.544	8.76	0.231	23.8*	0.627
PAF-P (full scale) δ=20° h/d=24 Port (Carreño, 2011)	4.63	37.9	0.075	1.76	62.7	16.93	20.9	0.552	15.0	0.397	93.1*	2.46
PAF-P (full scale) δ=20° h/d=2.2 Port (Carreño, 2011)	4.32	37.9	0.070	1.66	65.8	20.15	12.0	0.317	8.08	0.213	23.8*	0.627
Ship C (model) δ=35° h/d=6 (Lee y Kijima 2006)	3.09	2.5	0.516	1.36	27	2.5	1.19	0.478	1.14	0.454	-	-
Ship C (model) δ=35° h/d=1.2 (Lee y Kijima 2006)	3.09	2.5	0.494	2.47	5	5	0.436	0.174	0.436	0.174	-	-
Wide beam (model) δ=35° h/d=17 (Yoshimura 1988)	0.626	2.9	0.063	0.344	21	4.78	1.96	0.677	1.72	0.591	4.22	1.45
Wide beam (model) δ=35° h/d=1.2 (Yoshimura 1988)	0.259	2.9	0.045	0.194	2	3.92	0.149	0.052	0.137	0.047	0.302	0.104
Conventional beam (model) δ=35° h/d=15.3 (Yoshimura 1988)	0.938	3.2	0.100	0.469	18	4.16	1.44	0.451	1.29	0.402	3.50	1.09
Conventional beam (model) δ=35° h/d=1.2 (Yoshimura 1988)	0.471	3.2	0.035	0.353	3	9.6	0.530	0.166	0.502	0.157	0.307	0.095

* for this result u'_0 was estimated taking the resulting speed of the numerical simulations as the normalization basis

Discussion of results

The present study reveals the variation of the PP of a vessel with non-conventional propulsion and with a wide beam configuration, whose manoeuvrability behavior in shallow water is denoted non-standard NS (Yoshimura & Sakurai, 1988). Despite the particulars of the described ship, it is important to remark that the trend most usually mentioned on the related references is here proven by implementing the three available dynamic formulae. This behavior consists of the approaching of the PP to the center of gravity when maneuvers in shallow water are executed (with respect to those of deep water), such as it was explained above.

An important issue exhibited in the research carried out on the RSPV's manoeuvrability was the predominance of a non-linear nature in its motion (Carreño, 2011), and this feature turns out to be an important reason why there is a significant discrepancy between the results obtained with formula III and those of formulae I and II, as the former has been derived from a linear model that may give a good approximation for some vessels (Fossen, 2011), but it is not appropriate for the one which is being analyzed here.

From table 5 several important observations can be distinguished in regard to the collected information: in all of the cases reported here it is evident that there is a decrease of x_p when the ship shifts deep water to shallow water, whatever it is the employed formula or the evaluated ship, thus proving the most common premise about the PP in shallow water; the previous annotation is complemented by the fact that the drift angle decreases and final speed undergoes less relative loss than in deep water, which characteristics are as well cited in the consulted documentation; it can be appreciated that the wide beam ship in deep water presents a PP that falls more forward than in the other models, which besides shows good agreement between the numerical results and the published practical information; finally, the existing shift of location of the PP due to the depth variation is more pronounced in the case of the wide beam vessel than in that of conventional beam, even if the result from formula III is considered which apparently the less accurate, and thereby this last feature should be considered the most important finding of this work since this behavior had not been reported heretofore.

Conclusions

Thanks to the results of the full-scale tests it has been possible to prove experimentally the effect that shallow water maneuvering has on the PP, that is, show that the smaller the depth is the more the PP moves backwards approaching to G. According to the reported results, this variation is present independently from either the type of propulsion or the ship shape.

It has been observed a more pronounced effect in the PP variation for a wide-beam ship through different evaluation methods.

Finally, it was shown the poor approximation level that the method derived from Nomoto model (Formula III) may offer. This is because of the non lineal nature of the RSPV's motion.

Nomenclature

Symbol	Meaning	Units
x_0, y_0, z_0	Fixed coordinates	m
x, y, z	Moving coordinates	m
G	Center of gravity	-
L	Length	m
d	Draft	m
B	Beam	m
U	Absolute speed	m/s, knots
u	Longitudinal velocity (surge)	m/s
v	Lateral velocity (sway)	m/s
r	Angular speed (yaw)	deg/s, rad/s
X	Total force in surge direction	N
Y	Total force in sway direction	N
N	Total moment in yaw	Nm
R	Steady turning radius	m
m	Original mass	kg
m_x	Added mass in x	kg
m_y	Added mass in y	Kg
I_{zz}	Original moment of inertia around z	kgm^2
J_{zz}	Added moment of inertia around z	kgm^2
F_m	Factor of maneuver	-
C_B	Block Coefficient	-
t	Thrust deduction	-
h	Depth	m
T_j	Actual thrust	kN
$T_{j,0}$	Nominal thrust	kN
U_0	Initial speed of the maneuver	m/s, knots
x_j	Longitudinal position of the propulsion pump	m
$X', Y', N', u', v', r', m', I', J'$	Non-dimensional forms of forces (or coefficients), velocities and inertia terms	-

Greek symbols

Symbol	Meaning	Units
β	Drift angle	°, rad
Ψ	Heading angle	°, rad
δ	Rudder/jet angle	°, rad

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